

MODEL REDUCTION IN INTEGRATED CONTROLS-STRUCTURES DESIGN

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It is the objective of this paper to present a model reduction technique developed for the integrated controls-structures design of flexible structures. Integrated controls-structures design problems are typically posed as nonlinear mathematical programming problems, where the design variables consist of both structural and control parameters. In the solution process, both structural and control design variables are constantly changing; therefore, the dynamic characteristics of the structure are also changing. This presents a problem in obtaining a reduced-order model for active control design and analysis which will be valid for all design points within the design space. In other words, the frequency and number of the significant modes of the structure (modes that should be included) may vary considerably throughout the design process [1,2]. This is also true as the locations and/or masses of the sensors and actuators change. Moreover, since the number of design evaluations in the integrated design process could easily run into thousands, any feasible order-reduction method should not require model reduction analysis at every design iteration. In this paper a novel and efficient technique for model reduction in the integrated controls-structures design process, which addresses these issues, is presented.

OBJECTIVE

- Develop a model reduction technique for use in the integrated controls-structures design.
 - >Address the problem of a changing structure: the number and frequency of the significant modes may vary.
 - >Address the problem of control system implementation: sensor and actuator locations and masses.
 - >Address computational efficiency issues.

The approach presented in this paper is first to use a first-order Taylor's Series approximation of the open-loop eigenvalues and eigenvectors with the aid of their respective analytical derivatives with respect to both the structural and control design variables. Then, evaluating the significance of each mode through cost measures related to its controllability and observability [3], the number and frequency of the significant modes at the nominal design point, as well as the number and frequency of modes that might become significant in a prescribed neighborhood of the nominal point, are determined using a worst-case scenario approach. If the current design is within the prescribed neighborhood of the nominal design, the modes identified in the above are used in the control design and analysis. However, if the current design is outside the neighborhood, a single-point order reduction is performed.

APPROACH

- Evaluate the significance of each mode through its controllability and observability cost measures.
- Use a first-order Taylor's Series approximation of the open-loop eigenvalues and eigenvectors in a prescribed "linear" neighborhood about a nominal design.
- Identify the number and frequency of modes that may become significant within a neighborhood of a nominal design using a worst-case scenario approach.
- Perform "single-point" model reduction for design points outside the "linear" neighborhood.

The equations of motion for a flexible structure, in state-space form, are shown below, where A , B , and C are the plant, the actuator influence, and the sensor influence matrices, respectively. The plant matrix, in general, is nonsymmetric and fully populated. For a large flexible structure, the order of the initial model can be in the thousands, which makes it unsuitable for design and analysis. The classical approach for reducing the size of the problem is to introduce a model reduction method to eliminate dynamics characteristics that are outside the bandwidth of interest, hence reducing the computational burden. This naturally leads to the question whether the problem can be reduced even further, i.e., are there modes within the bandwidth that do not contribute much to the dynamic response? In order to distinguish a significant mode from an insignificant mode, a measure of modal significance must be adopted and compared. In this paper, the controllability and observability cost measures presented in [3] are used.

CONTROLLABILITY AND OBSERVABILITY COST MEASURES

The equations of motion, in state-space form, are given as:

$$\dot{q} = Aq + Bu$$

$$y = Cq$$

A measure of controllability and observability may be defined as:

$$\alpha_{ci} = \frac{\phi_i^H W_c \phi_i}{\phi_i^H \phi_i}$$

$$\alpha_{oi} = \frac{\phi_i^H W_o \phi_i}{\phi_i^H \phi_i}$$

Where α_{ci} and α_{oi} are measures of the closeness of the i^{th} mode w.r.t. the controllable (observable) range spaces, defined by $\Re(W_c)$ ($\Re(W_o)$).

Transforming the equations of motion from physical coordinates to modal coordinates results in a plant matrix that is block diagonal. Normalizing the modes of the structure for unity modal mass results in 2×2 blocks of the form shown below. Due to this particular block-diagonal nature of A , its eigenvectors have a special form as well, such that there are complex conjugate vector pairs associated with each 2×2 block. This considerably simplifies the expressions for controllability and observability cost measures.

CONTROLLABILITY AND OBSERVABILITY COST MEASURES (CONT'D)

- If the modes of the structure are normalized to produce unity modal mass, i.e.,

$$X_i^T M X_j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

- The plant matrix can be written in modal form as:

$$A = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & A_n \end{bmatrix}$$

- A_i 's have a 2x2 diagonal block form:

$$A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\xi_i\omega_i \end{bmatrix}$$

Because of this special form of the eigenvectors of A , it becomes apparent that the controllability and observability cost measures do not require full multiplication of the matrices and vectors, but rather may be reduced to a series of 2×1 vector and 2×2 matrix multiplications. The components of the 2×1 vector are the two nonzero components of the eigenvectors of A , and the 2×2 matrices are block diagonal components of the controllability and observability grammians. In other words, the modal controllability and observability cost measures decouple, i.e., they depend only on the corresponding modal parameters.

CONTROLLABILITY AND OBSERVABILITY COST MEASURES (CONT'D)

- Simplified expressions for the controllability and observability cost measures are given as:

$$\alpha_{ci} = \frac{1}{1 + \omega_i^2} \begin{Bmatrix} 1 \\ \xi_i \omega_i + \omega_i \sqrt{1 - \xi_i^2} \underline{j} \end{Bmatrix}^H W_{ci} \begin{Bmatrix} 1 \\ \xi_i \omega_i + \omega_i \sqrt{1 - \xi_i^2} \underline{j} \end{Bmatrix}$$

$$\alpha_{oi} = \frac{1}{1 + \omega_i^2} \begin{Bmatrix} 1 \\ \xi_i \omega_i + \omega_i \sqrt{1 - \xi_i^2} \underline{j} \end{Bmatrix}^H W_{oi} \begin{Bmatrix} 1 \\ \xi_i \omega_i + \omega_i \sqrt{1 - \xi_i^2} \underline{j} \end{Bmatrix}$$

- W_{ci} and W_{oi} represent the i^{th} 2×2 block on the diagonal of the grammians W_c and W_o , respectively.

The 2×2 block diagonal elements of the controllability and observability grammians may be obtained analytically. Here, the vectors ψ_i , γ_{di} , and γ_{ri} represent the input, displacement output, and rate output influence coefficients, respectively. These solutions may then be combined with the reduced expressions for the controllability and observability cost measures yielding simplified modal cost measures, α_{ci} and α_{oi} , as shown below.

CONTROLLABILITY AND OBSERVABILITY COST MEASURES (CONT'D)

- Simplified controllability and observability grammians:

$$W_{ci} = \begin{bmatrix} \frac{1}{4\xi_i\omega_i^3} & 0 \\ 0 & \frac{1}{4\xi_i\omega_i} \end{bmatrix} \psi_i^T \psi_i$$

$$W_{oi} = \begin{bmatrix} \frac{1+4\xi_i^2}{4\xi_i\omega_i^3} \gamma_{di}^T \gamma_{di} + \frac{\omega_i}{4\xi_i} \gamma_{ri}^T \gamma_{ri} & \frac{1}{2\omega_i^2} \gamma_{di}^T \gamma_{di} \\ \frac{1}{2\omega_i^2} \gamma_{di}^T \gamma_{di} & \frac{1}{4\xi_i\omega_i} \gamma_{ri}^T \gamma_{ri} + \frac{1}{4\xi_i\omega_i^3} \gamma_{di}^T \gamma_{di} \end{bmatrix}$$

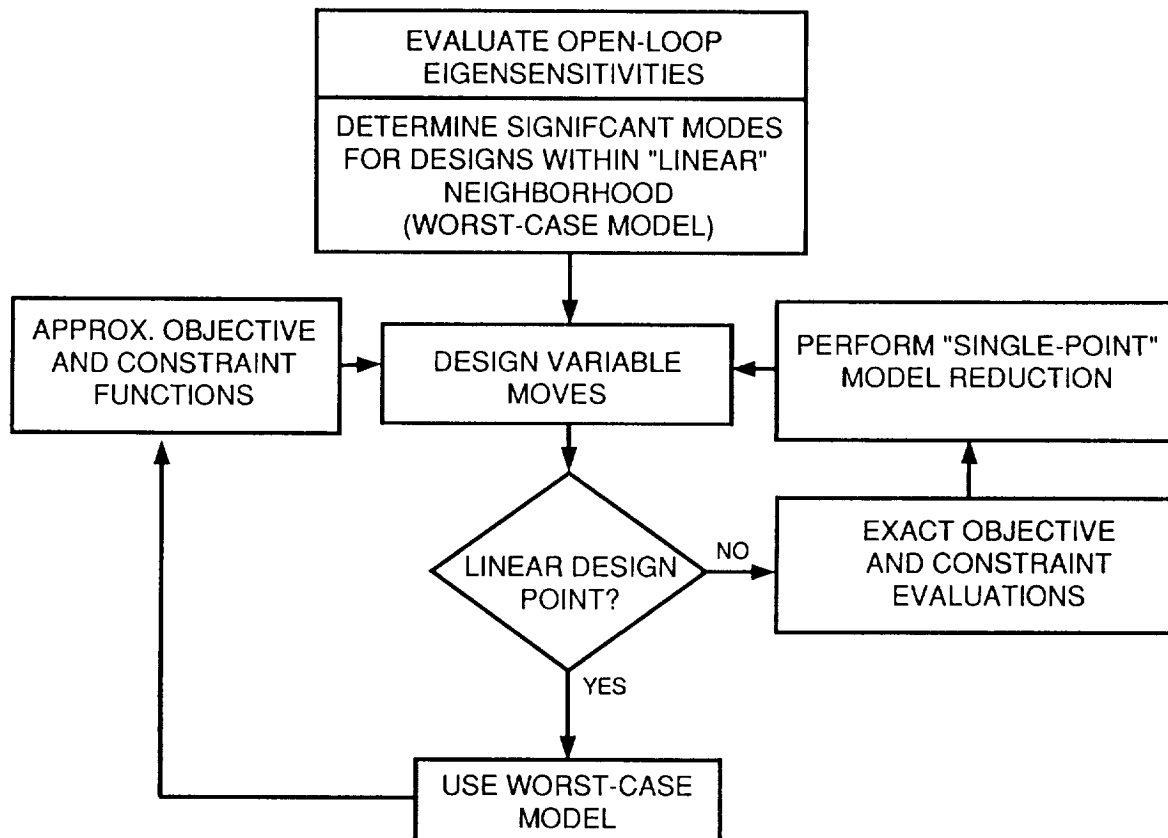
- Simplified modal controllability and observability cost measures:

$$\alpha_{ci} = \frac{1}{4(1 + \omega_i^2)\xi_i} \left\{ \frac{1}{\omega_i^3} + \omega_i \right\} \psi_i^T \psi_i$$

$$\alpha_{oi} = \frac{1}{2(1 + \omega_i^2)\xi_i} \left\{ \frac{1 + 4\xi_i^2}{\omega_i} \gamma_{di}^T \gamma_{di} + \omega_i \gamma_{ri}^T \gamma_{ri} \right\}$$

The model reduction algorithm computes the sensitivity of the open-loop eigenvalues and eigenvectors each time that the optimization requests gradient information (gradient of the objectives and constraints with respect to the design variables). Then, upper bound values for the modal controllability and observability cost measures α_{ci}^U and α_{oi}^U are computed and compared with preset threshold values in order to identify the significant modes for designs within the prescribed neighborhood of the nominal design. Now, if an upcoming design is within this neighborhood, these identified modes are used to form a design model for control synthesis. However, if an upcoming design is outside this neighborhood, a single-point model reduction is performed to identify the significant modes for control design. This process is repeated until the integrated design optimization converges.

MODEL REDUCTION ALGORITHM



Consider the real symmetric structural eigenvalue problem, as stated below, where K and M are symmetric positive semi-definite stiffness and symmetric positive-definite mass matrices, respectively. Differentiating the defining eigenvalue problem with respect to a structural design variable, ρ_j , gives expressions for both the eigenvalue and eigenvector derivatives. Premultiplying by the eigenvector yields a simple expression for the eigenvalue derivative. However, due to the rank deficiency of the defining eigenvalue problem, the eigenvector derivative cannot be uniquely determined from this expression.

STRUCTURAL EIGENSYSTEM (OPEN-LOOP) SENSITIVITY ANALYSIS

- Structural Eigenvalue Problem: $(K - \lambda_i M)X_i = 0$
- Eigenvalue Derivative: $\frac{\partial \lambda_i}{\partial \rho_j} = X_i^T \left(\frac{\partial K}{\partial \rho_j} - \lambda_i \frac{\partial M}{\partial \rho_j} \right) X_i$
- Eigenvector Derivative:

$$(K - \lambda_i M) \frac{\partial X_i}{\partial \rho_j} = \frac{\partial \lambda_i}{\partial \rho_j} M X_i - \left(\frac{\partial K}{\partial \rho_j} - \lambda_i \frac{\partial M}{\partial \rho_j} \right) X_i$$

Note that $(K - \lambda_i M)$ is rank deficient.

Expressing the eigenvector derivative as a linear combination of all the eigenvectors and substituting it into the defining eigenvector derivative equation gives an expression for the particular solution, V_{ij} . Noting that the particular solution is mass-orthogonal with respect to the eigenvector provides a set of linear constraints that may be used to eliminate the singularity problems of the unconstrained expression. The constant, C_{ij} , may be obtained by differentiating the eigenvector normalization condition $X_i^T M X_i = 1$. For a detailed development of the eigensystem sensitivity equations, see [4].

STRUCTURAL EIGENSYSTEM SENSITIVITY ANALYSIS (CONT'D)

A solution for the eigenvector derivative may be obtained by expressing it as a linear combination of all of the eigenvectors

$$\frac{\partial X_i}{\partial \rho_j} = \sum_{\substack{k=1 \\ k \neq i}}^n C_{kj} X_k + C_{ij} X_i = V_{ij} + C_{ij} X_i$$

Where V_{ij} and C_{ij} are defined as follows:

$$\begin{bmatrix} (K - \lambda_i M) & M X_i \\ X_i^T M & 0 \end{bmatrix} \begin{Bmatrix} V_{ij} \\ \frac{\partial \lambda_i}{\partial \rho_j} \end{Bmatrix} = \begin{Bmatrix} -\left(\frac{\partial K}{\partial \rho_j} - \lambda_i \frac{\partial M}{\partial \rho_j}\right) X_i \\ 0 \end{Bmatrix}$$

$$C_{ij} = -\frac{1}{2} X_i^T \frac{\partial M}{\partial \rho_j} X_i$$

Both modal controllability and observability cost measures, α_{ci} and α_{oi} , are functions of the design variables ρ . As the current design moves away from the nominal design point, the number and frequency of the significant modes measured by α_{ci} and α_{oi} might change. Consequently, if upper bound values, α_{ci}^U and α_{oi}^U , can be established for the modal controllability and observability cost measures for design points within a prescribed neighborhood of a nominal design, they may be used to identify the modes that are currently significant and modes that might become significant as the design optimization progresses.

MODAL COST APPROXIMATIONS

- Compute upper bound values for the controllability and observability cost measures if the new design is within the neighborhood of the nominal design:

—>Find an upper bound value for the controllability cost measure:

$$\max_{\rho} \left\{ \alpha_{ci}(\rho) = \frac{1}{4(1 + \omega_i^2(\rho))\xi_i} \left\{ \frac{1}{\omega_i^3(\rho)} + \omega_i(\rho) \right\} \psi_i^T(\rho) \psi_i(\rho) \right\}$$

—>Find an upper bound value for the observability cost measure:

$$\max_{\rho} \left\{ \alpha_{oi}(\rho) = \frac{1}{2(1 + \omega_i^2(\rho))\xi_i} \left[\frac{1 + 4\xi_i^2}{\omega_i(\rho)} \gamma_{di}^T(\rho) \gamma_{di}(\rho) + \omega_i(\rho) \gamma_{ri}^T(\rho) \gamma_{ri}(\rho) \right] \right\}$$

Upper bound values for the modal controllability and observability cost measures may be established by using a worst-case scenario approach, wherein the maximum possible contribution from each term is used in the computations. These terms involve functions of the open-loop eigenvalues and the input and the output influence vectors which are approximated by a first-order Taylor's Series expansion.

MODAL COST APPROXIMATION (CONT'D)

- Obtain upper bound values for the modal controllability and observability cost measures

$$\alpha_{ci}(\rho) \leq 1/(4\xi_i) \left[\max_{\rho} \left\{ 1/\omega_i^3(\rho) (1 + \omega_i^2(\rho)) \right\} + \max_{\rho} \left\{ \omega_i(\rho) / (1 + \omega_i^2(\rho)) \right\} \right] \cdot \max_{\rho} \left\{ \psi_i^T(\rho) \psi_i(\rho) \right\} \equiv \alpha_{ci}^U$$

$$\alpha_{io}(\rho) \leq \left[(4\xi_i^2 + 1) / (2\xi_i) \right] \left\{ \left\{ \max_{\rho} \left[1/\omega_i(\rho) (1 + \omega_i^2(\rho)) \right] \max_{\rho} \left\{ \gamma_{di}^T(\rho) \gamma_{di}(\rho) \right\} \right\} \right\} + \left[1 / (2\xi_i) \right] \left\{ \left\{ \max_{\rho} \left[\omega_i(\rho) / (1 + \omega_i^2(\rho)) \right] \max_{\rho} \left\{ \gamma_{ri}^T(\rho) \gamma_{ri}(\rho) \right\} \right\} \right\} \equiv \alpha_{oi}^U$$

- Use a first-order Taylor's Series approximation for the open-loop eigenvalues and the influence coefficients

$$\omega_i(\rho) \approx \omega_i(\rho_o) + \sum_{j=1}^{n_d} \frac{\partial \omega_i}{\partial \rho_j} \bigg|_{\rho_{oj}} (\rho_j - \rho_{oj})$$

$$\psi_i(\rho) \approx \psi_i(\rho_o) + \sum_{j=1}^{n_d} \frac{\partial \psi_i}{\partial \rho_j} \bigg|_{\rho_{oj}} (\rho_j - \rho_{oj})$$

Upper bound values for terms in the modal controllability and observability cost measures that involve the influence vectors ψ_i , γ_{di} , and γ_{ri} may be obtained by evaluating these terms at a design point in the direction of the steepest ascent and at the boundary of the neighborhood. Here, it is assumed that the coupling between the influence vectors corresponding to different modes is small. The remaining terms in the cost measures involve functions of the open-loop eigenvalues. All but one of these functions of ω_i have no maximum. Only the function $f^* = \omega_i(\rho)/(1 + \omega_i^2(\rho))$ has a maximum at $\omega_i(\rho) = 1$. Consequently, upper bound values for all these functions except f^* can be obtained by computing the maximum value of these functions at the smallest and the largest possible values of ω_i (ω_i^L and ω_i^U) within the prescribed neighborhood. As for f^* , if $\omega_i(\rho) = 1$ is within the prescribed neighborhood, then $f_{max}^* = 1/2$. Otherwise, the same procedure as for other functions is used.

MODAL COST APPROXIMATION (CONT'D)

- Upper bound value for the influence coefficient terms:

$$\begin{aligned} \max_{\rho} \{ \psi_i^T(\rho) \psi_i(\rho) \} &\approx \psi_i^T(\rho_o) \psi_i(\rho_o) + 2 \sum_{j=1}^{n_d} \psi_i^T(\rho_o) [\partial \psi_i / \partial \rho_j] \text{sgn} \{ \psi_i^T(\rho_o) [\partial \psi_i / \partial \rho_j] \} \rho_{oj} \epsilon + \\ &\sum_{j=1}^{n_d} \sum_{k=1}^{n_d} [\partial \psi_i / \partial \rho_j]^T [\partial \psi_i / \partial \rho_k] \text{sgn} \{ \psi_i^T(\rho_o) [\partial \psi_i / \partial \rho_j] \} \text{sgn} \{ \psi_i^T(\rho_o) [\partial \psi_i / \partial \rho_k] \} \rho_{oj} \rho_{ok} \epsilon^2 \end{aligned}$$

- Upper bound values for the scalar functions of ω_i may be obtained by computing these functions at ω_i^U and ω_i^L .

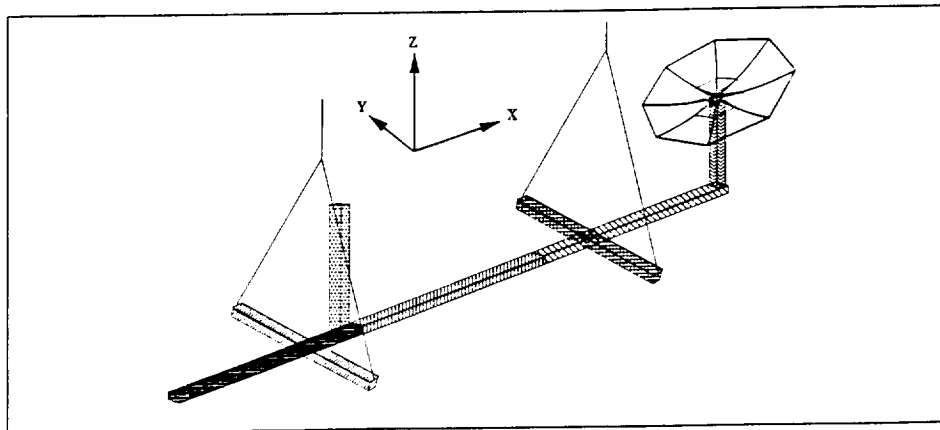
$$\begin{aligned} \omega_i^L(\rho) &= \omega_i(\rho_o) - \sum_{j=1}^{n_d} |\partial \omega_i / \partial \rho_j| \rho_{oj} \epsilon \\ \omega_i^U(\rho) &= \omega_i(\rho_o) + \sum_{j=1}^{n_d} |\partial \omega_i / \partial \rho_j| \rho_{oj} \epsilon \end{aligned}$$

The CSI Evolutionary Model is a laboratory testbed designed and constructed at the NASA Langley Research Center for experimental validation of the control design methods and the integrated design methodology [5]. The Phase-Zero Evolutionary Model, shown in the figure, consists of a 62-bay central truss, with each bay 10 inches long, two vertical towers, and two horizontal booms. The structure is suspended using two cables as shown. A laser source is mounted at the top of one of the towers, and a reflector with a mirrored surface is mounted on the other tower. The laser beam is reflected by the mirrored surface onto a detector surface 660 inches above the reflector. Eight proportional, bi-directional, gas thrusters provide the input actuation, while collocated servo accelerometers provide output measurements. An integrated controls-structures design of this test article is sought.

To perform the integrated design, the structure was divided into seven sections, three sections in the main bus, and one section each for the two horizontal booms and two vertical towers. Three structural design variables were used in each section, namely, effective cross-sectional area of the longerons, the battens, and the diagonals, making a total of 21 structural design variables.

STRUCTURAL DESIGN VARIABLES

- Structure is divided into seven sections
- The effective cross-sectional areas of longerons, battens and diagonals are chosen as design variables
- Total of 21 structural design variables



The static (or constant-gain) dissipative controller which employs collocated and compatible actuators and sensors, and consists of feedbacks of the measured attitude vector y_p and the attitude rate vector y_r using constant, positive-definite gain matrices G_p and G_r , is used for feedback control. This controller is robust in the presence of parametric uncertainties, unmodelled dynamics, and certain types of actuator and sensor nonlinearities [6]. However, the performance of such controllers is inherently limited because of their structure. Here, two of the eight available actuators were used to generate persistent white-noise disturbances, while the remaining six actuators were used for feedback control. The static dissipative controller uses a 6 x 6 diagonal rate-gain matrix with no position feedback (since this system has no zero-frequency eigenvalues, position feedback is not necessary for asymptotic stability). Thus, in the integrated design with the static dissipative controller, the total number of design variables was 27 (21 structural plus 6 control design variables).

CANDIDATE CONTROLLERS

Static Dissipative Controllers

$$u = -G_r y_r$$

- Collocated sensors and actuators
- Positive definite gain matrices
- Robust in presence of model uncertainties
- May have limited performance
- Elements of the Cholesky-factor matrix of the rate gain matrix are used as control design variables (no position feedback)

$$G_r = L_r L_r^T$$

An integrated controls-structures design was obtained by minimizing the steady-state average control power in the presence of white-noise input disturbances with unit intensity (i.e., standard deviation intensity = 1 lbf.) at actuators No. 1 and 2 (located at the end of the main bus nearest to the laser tower). A constraint was placed on the steady-state rms position error at the laser detector (above the structure) for reasonable steady-state pointing performance. Additionally, the total mass of the structure was constrained to facilitate a fair comparison with the phase-0 design. The six remaining actuators were used in the control design, along with velocity signals (required for feedback by the dissipative controllers) obtained by processing the accelerometer outputs. Side constraints were also placed on the structural design variables for safety and practicality concerns. Lower bound values were placed on these variables to satisfy structural integrity requirements against buckling and stress failures. On the other hand, upper bound values were placed on these variables to accommodate design and fabrication limitations.

DESIGN PROBLEM

- Pose the integrated controls-structures design as a simultaneous optimization problem
- Minimize the average control power

$$J \equiv \text{Trace} \left\{ E \left\{ uu^T \right\} \right\}$$

subject to

$$\text{Trace} \left\{ E \left\{ y_{los} y_{los}^T \right\} \right\} \leq \epsilon$$

$$M_{tot} \leq M_{budget}$$

- Side constraints on the structural design variables to accommodate safety, reliability, and fabrication issues

The controls-structures integrated design results are shown below. The results indicate that the evaluation (fifty-mode) model and reduced-order model converged to essentially the same final design. This is a clear indication that the model reduction method presented in this paper can handle possible discontinuities associated with the changing dynamic characteristics of the evolving structure.

The controls-structures integrated design results were obtained using the Automatic Design Synthesis (ADS) software package [7]. All solutions were computed using an interior penalty function method with a Broyden-Fletcher-Goldfarb-Shanno method for the unconstrained subproblem.

INTEGRATED DESIGN RESULTS

	CONTROL POWER	RMS POINTING	TOTAL MASS
EVAL. MODEL	2.64	2.999	1.896
REDUCED-ORDER MODEL	2.57	2.998	1.918

The table below presents a computational performance comparison of the evaluation model and reduced-order model. The results indicate that the model reduction scheme yielded approximately a 49 percent reduction in CPU time. This increased performance can be attributed to CPU time reductions in both the closed-loop analysis, as well as those gained by introducing open-loop eigenvalue/vector approximations. It should also be noted that the model reduction method required 8 percent more function evaluations to obtain an optimal design. This may be attributed to inaccuracies induced by the open-loop eigensystem approximations.

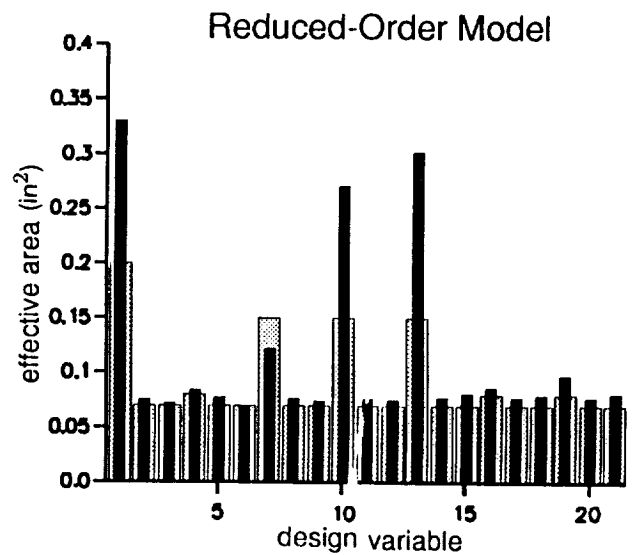
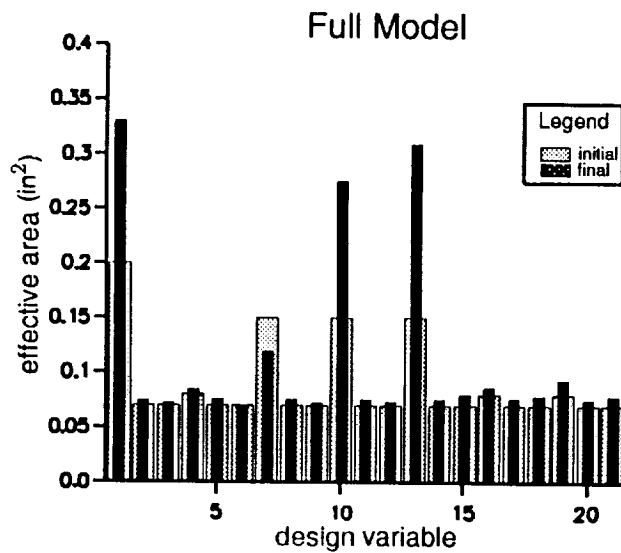
COMPUTATIONAL REQUIREMENTS

	CPU TIME * (TOTAL)	CPU TIME (AVG. PER EVALUATION)
EVAL. MODEL	28 hrs. 19 min.	82.7 sec.
REDUCED-ORDER MODEL	14 hrs. 30 min.	38.9 sec.

* SUN SPARC 2 workstation.

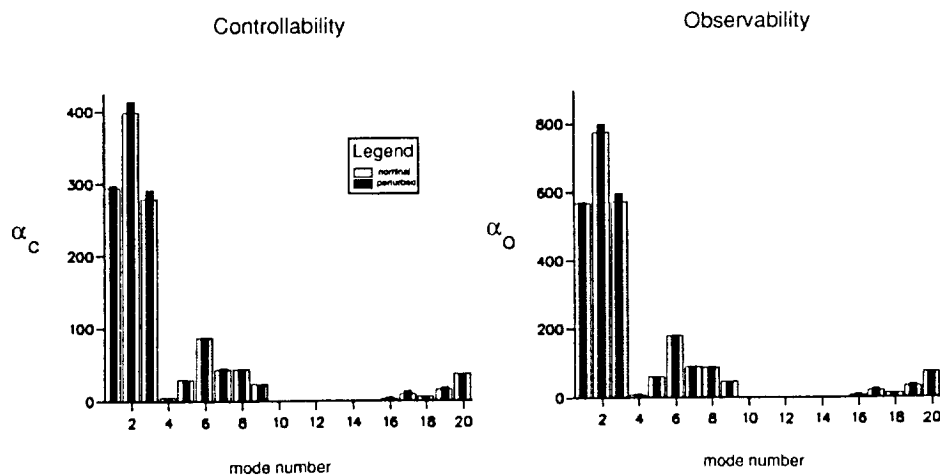
The bar charts shown below present the resulting structural design variables for both the full model and the reduced-order model in terms of initial versus final design. The results indicate that the two methods converged to basically the same final design.

STRUCTURAL DESIGN VARIABLES



In the bar charts below, controllability and observability cost measures for the first 20 modes are listed. The controllability and observability cost measures are for a nominal point and the worst-case values within a 5 percent perturbation from the nominal. Using the worst-case scenario approach described earlier, the number of modes retained for closed-loop analysis was increased from 36 to 38. This chart also indicates the relative sensitivity of α_{ci} and α_{oi} with respect to changes in the structural design variables. It can be observed that the first three suspension modes (1–3) are the most controllable and observable modes. However, the last two modes (5 and 6) along with the first three flexible modes (7–9) are quite controllable and observable as well. Moreover, it can be seen that modes that are not significant at the nominal design point (modes 17 and 19) are as sensitive to design perturbations as lower frequency modes, and, therefore, might become significant as the design optimization progresses. Although not shown, the same level of sensitivity was found in modes 21 through 50. It should be noted that in this design problem the sensors and actuators are collocated, thereby producing values for the controllability and observability cost measures which are similar, but different in scaling.

CONTROLLABILITY AND OBSERVABILITY COST MEASURES



A novel and efficient method for model order reduction in the integrated controls-structures design process has been developed. The method uses a linear approximation of the open-loop eigenvalues and eigenvectors and identifies, through a worst-case scenario, the structural modes that are significant at a nominal design point along with modes that might become significant as the optimization moves the structural design variables within a prescribed neighborhood of the nominal design point. Consequently, this approach can handle the discontinuities that may hamper the integrated design optimization process because of the evolving structure, i.e., the frequency and number of significant structural modes can change at each design iteration. Although in this paper modal controllability and observability cost measures were used to evaluate the significance of each mode for inclusion in the control design model, the approach of linear approximation and worst-case analysis can be used in conjunction with other modal cost measures as well. Finally, further research is required to identify proper threshold levels for controllability and observability cost measures as well as to choose the size of the prescribed neighborhood used in the linear approximation.

CONCLUDING REMARKS

- A new and efficient method for model order reduction in the integrated controls-structures design has been developed.
 - >The method can handle the discontinuity problems that may hamper the optimization process.
 - >The method can be used in conjunction with other model reduction techniques.
- Further research is required in choosing the threshold levels for controllability and observability, as well as the size of the neighborhood for linear approximation.

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